AP Calculus AB

Summer Packet

Complete the following exercises throughout the summer. Do not wait until the week before school to start! The skills and concepts represented in this packet have been part of your Algebra 2 and Pre-Calculus courses. It is expected you know how to do every problem in this packet. We will spend one day going through questions in class in the fall. Expect a quiz on this material.

Mrs. Iszczyszyn

Mr. Friedman

Section 1 – Types of Graphs

Identify the transformations in the following functions:

1. $f(x) = \sqrt{2x - 3}$

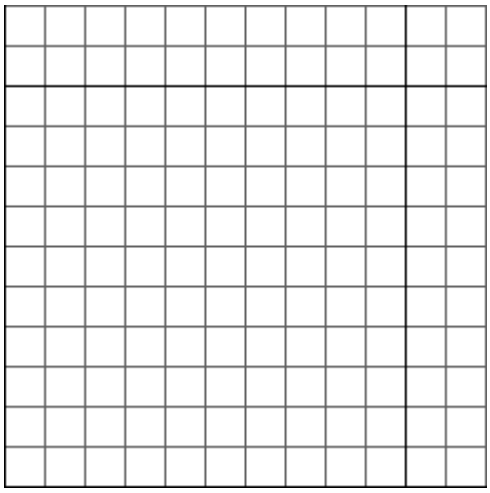
2. $g(x) = (x - 2)^2 + 1$

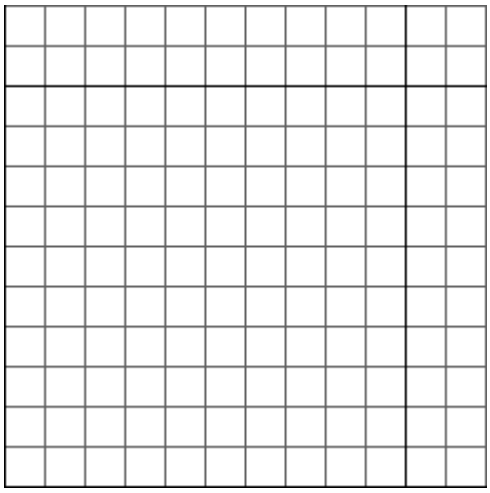
3. $h(x) = -(x - 12)^4 - 3$

Graph the following functions on the graphs provided below WITHOUT the use of a Calculator:

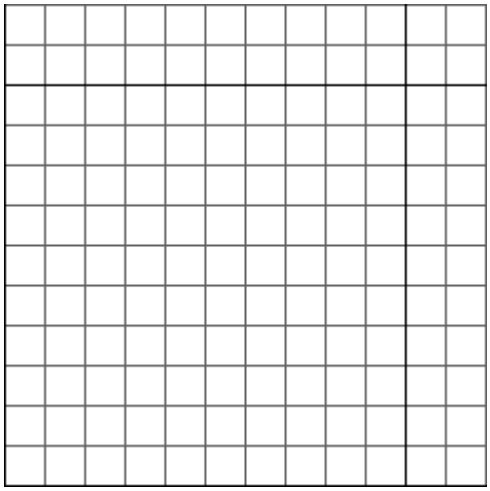
Basic Functions

4. Graph: $f(x) = \mathbf{e}^x$ and $g(x) = \mathbf{e}^{-x}$

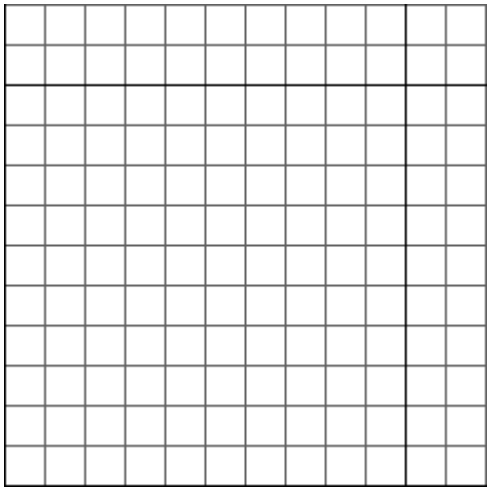




5. Graph: $f(x) = \ln(x)$

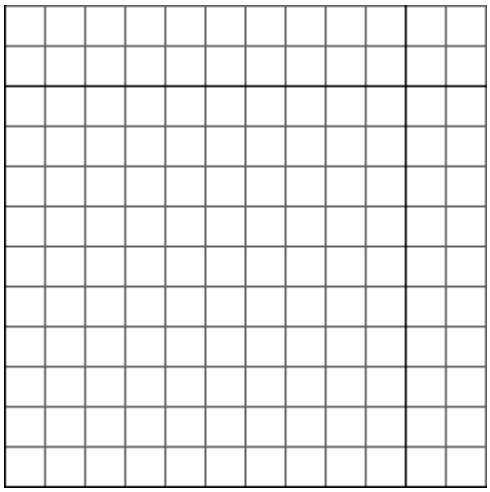


6. Graph: $f(x) = |x|$



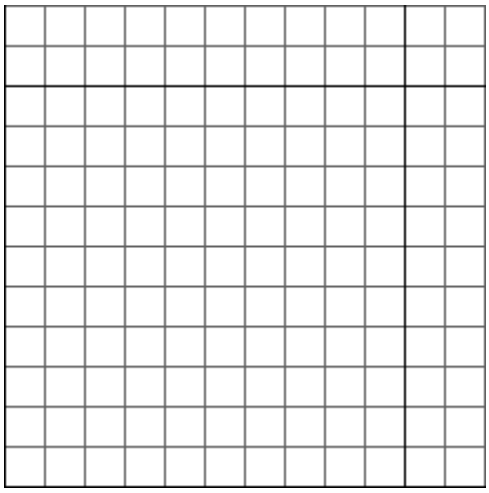
7. Square Root Functions:

$y=\sqrt{(x+2)}$



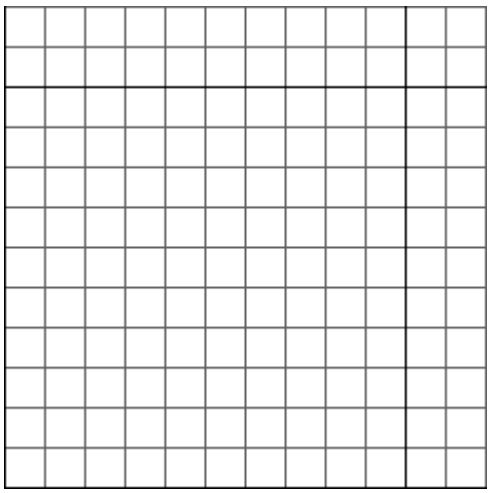
8. Cubic Functions:

$y = x^3 + 3x^2 + x$



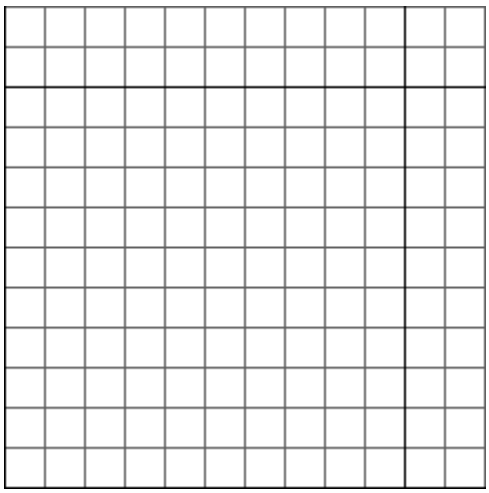
9. Absolute Value Functions:

$y = | x^2 - 3x - 4 |$



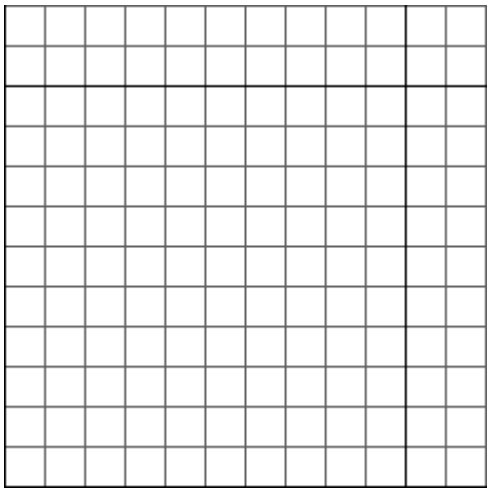
10. Exponential Functions:

$y = 3^x + 4$



11. Piecewise Functions:

$$f(x)=\begin{cases}2x^2-1, & x<1\\ x+4, & x\geq 1\end{cases}$$



Write the equation of the function according to each condition if f(x) = e^x

12. Translate left 4 units and down 2

13. Translate right 3 units and stretch vertically by 3

14. So that f(0) = 3

Section 2 – Functions

Use the table to find the following composite functions.

f(5)	*g(6)*	*h(6)*
6	5	4

15. $f(g(6))$

16. $h(f(g(6)))$

f(x)	*g(x)*	*h(x)*	*j(x)*
$x^2 + 2x$	$\sqrt{x}$	$x^2 + 5x + 8$	$\frac{x+1}{x^2}$

17. $f(j(1))$

18. $f(g(x))$

19. $f(g\left(h(j(1))\right))$

Find the following inverse functions.

20. Find the inverse of $f(x) = 4x + 12$

21. Find $f^{-1}(x)$ if $f(x) = \ln|x| + 7$

Identify if the functions are even/odd/neither

22. $f(x) = 2x^4 - 7x^2 + 5$

23. $g(x) = \sin(2x)$

Section 3 – Characteristics of Rational Functions

Without graphing the function, state the asymptotes and the domain/range of the function.

24. $y = \frac{-2x^2+1}{2x^3+4x^2}$

25. $y = \frac{x}{(x-1)(x+2)}$

26. $y = \frac{5}{(x+2)^2}$

27. $y = \frac{2x+4}{x-1}$

Let r(x) = f(x)/g(x) be a rational function where f(x) = 8x + 3 and g(x) is either linear or quadratic.

28. Choose **g(x)** so that **r(x)** has one horizontal at **y = 2** and one vertical asymptote at **x = 0**.

29. Choose **g(x)** so that **r(x)** has one horizontal at **y = 0** and two vertical asymptotes at **x = -3** and **x = 3**, respectively.

30. Choose **g(x)** so that **r(x)** has one horizontal at **y = 0** and one vertical asymptote at **x = 0**.

31. Choose **g(x)** so that **r(x)** has one horizontal at **y = 1** and one vertical asymptote at **x = 1**.

Find the x-intercepts

32. $f(x) = (x+4)(x+2)(x-1)$

Find the y-intercept

33. $f(x) = \frac{x^2-x-6}{x^2-1}$

Identify the hole in the rational function, written as an ordered pair.

34. $f(x) = \frac{x^2+2x-8}{x^2-x-2}$

Determine the left and right-hand behavior of the graph

35. $f(x) = -x^3 + 4x$

Section 4 – Properties of Exponents / Logarithmic and Exponential Functions

Simplify the following expressions involving exponents.

36. $(2a^{12}b^3)(3a^2b^4)$

37. $\left(\frac{3x^4y^{-3}z^2}{4x^{-3}y^{10}z}\right)^2$

38. $\left(\frac{b^3\sqrt{5b+2}}{a-b}\right)^2$

Rewrite as an equivalent expression

39. $2\ln(e^2)$

40. $\log_5 125$

41. $\log_4 \frac{1}{2}$

42. $\log 1000000$

43. $\log_b 1$

44. $\ln e^x$

45. $\log_{10}\sqrt{10}$

46. $\frac{1}{2}\log x + \log y - 3\log z$

47. $\log_3 81 + \log 0.001$

48. $\log_4 16 + \log_4 64$

49. $\log_x x^2 + \log_x x^3$

Suppose x = log(A) and y = log(B), write the following expressions in terms of x and y.

50. $\log(AB) =$

51. $\log(A)\log(B) =$

52. $\log\left(\frac{A}{B^2}\right) =$

Determine which functions (if any) are equivalent (without a calculator):

53. $f(x) = 3^{x-2}$ $g(x) = 3^x - 9$ $h(x) = 3^x / 9$

54. $f(x) = 5^{-x} + 3$ $g(x) = 5^{3-x}$ $h(x) = -5^{x-3}$

Using either the model given in the problem or a model you create, answer the following problems.

55. A recent study revealed that the amount of time a person spent working on math over the summer directly affected the grade they received on their first test. This relationship can be modeled by the equation $G = 5e^{kt}$ where G is the grade, t is the number of hours the person spent working on math over the summer, and k is a constant.

a) Suppose a student spent 8 hours on math over the summer and a grade of 75 is earned on the first test. What is the value of k?

b) Using the same value of k, determine the amount of hours one should spend studying in order to earn a 90 on the first test.

56. A researcher collects data on rabbit population over a 22-month period. The population (in thousands) is given in the table below:

Month	0	2	4	6	8	10	12	14	16	18	20	22
Number	10	12	14	16	22	30	35	39	44	48	50	51

a) Draw a scatter-plot of the data and find a logistic regression model.

b) What can you conclude about the limit of the rabbit population growth in this area? Justify.

Section 5 – Simplifying Algebraic Expressions

Simplify each expression. Leave your answer in its most practical form. [Hint: only expand if it is useful in simplifying the problem]

57. $(3x-3)(x^4+2x^3)+(4x^3+6x^2)(x^2-3x+2)$

58. $\frac{1}{4}(3x^2+2x)^{-2/3}(3x+2)$

59. $\dfrac{1+\dfrac{1}{x^2}}{\dfrac{x}{1}-1}$

60. $3x^0(x^3+2)^{-2}+4x^2(3(x^3+2))^{-3}$

61. $3x^3(3x-1)^{-1}+2x$

Section 6 – Trigonometry

Prove the following.

62. $tanxsinx + cosx = secx$

63. $sinx - sinxcos^2x = sin^3x$

Convert to Degrees

64. $\frac{17\pi}{6}$

65. 1.4

Convert to Radians

66. 200º

67. 120º

Find the exact value of the following. You will need to be able to know these cold.

68. sin30º

69. cos2π

70. tan45º

71. $\cot\frac{5\pi}{4}$

72. sin120º

73. $\sin\frac{\pi}{3}$

74. $\cos\frac{7\pi}{6}$

75. $\tan\frac{7\pi}{6}$

76. $\sec\frac{11\pi}{6}$

77. $\cot 0$

78. $\csc\frac{21\pi}{4}$

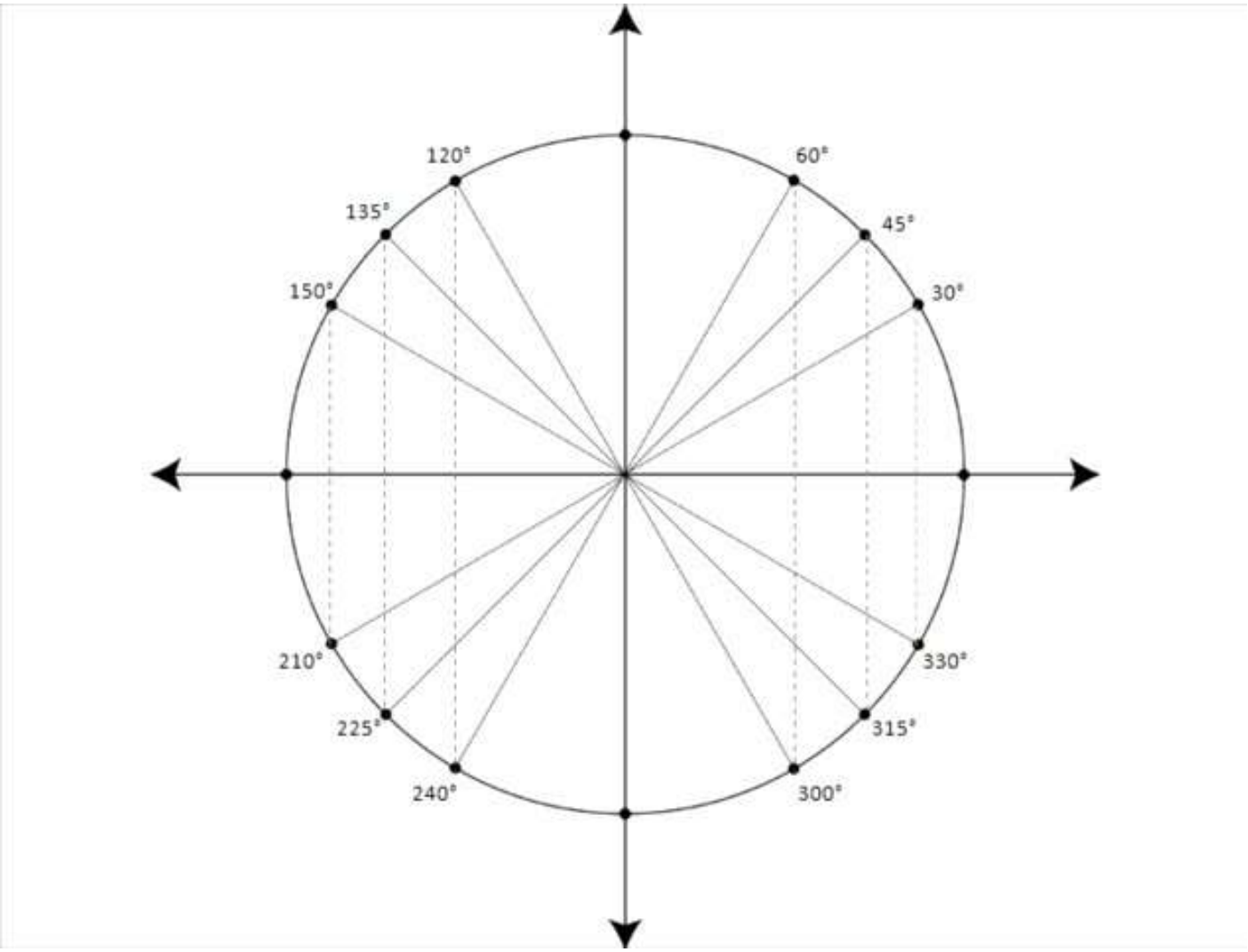
79. $\tan\frac{\pi}{4} + \sin\pi$

80. $\tan^{-1}\sqrt{3}$

81. $\sin^2(4x-5) + \cos^2(4x-5)$

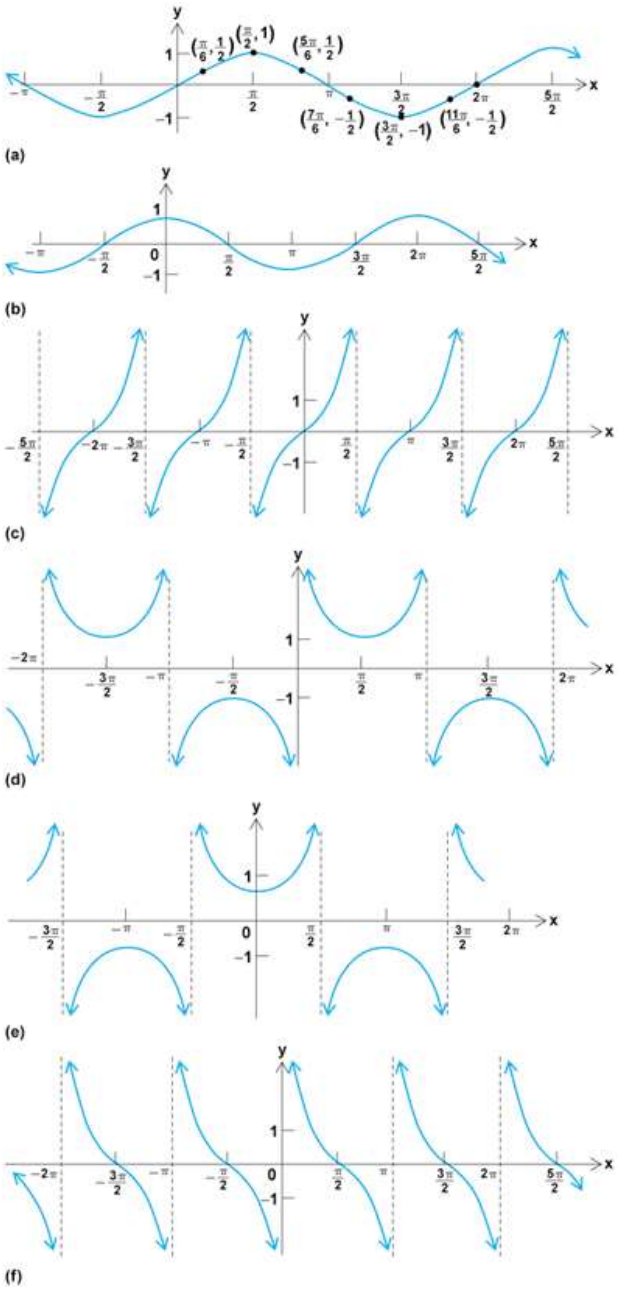
Label on the following unit circle:

82. the angle measurements in radians
the sine of each angle and the cosine of each angle as an ordered pair



83. Match each function to the corresponding graph.

i. $y = \tan(x)$
ii. $y = \sec(x)$
iii. $y = \cos(x)$
iv. $y = \sin(x)$
v. $y = \csc(x)$
vi. $y = \cot(x)$



84. If the amplitude of a sinusoidal function is doubled, does the period change? Justify.

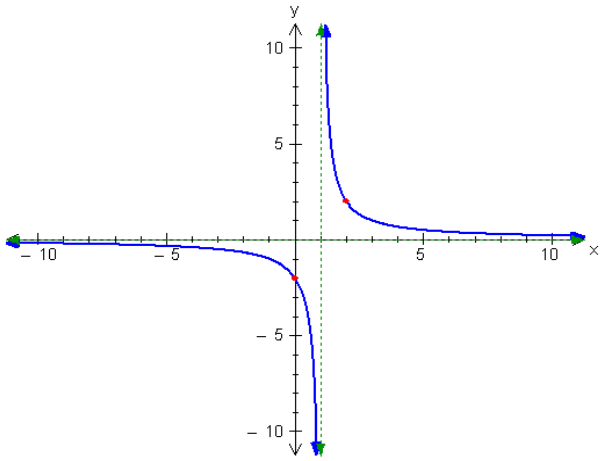
Section 7 – Limits

Evaluate the following limits.

85. $\lim_{x\to 0} \frac{2-x}{x^2+4}$

86. $\lim_{x\to 4} f(x)$ such that $f(x) = \begin{cases} 3x-2, & x \neq 4 \\ 15, & x = 4 \end{cases}$

87. The following is a graph of f(x)



a) $\lim_{x\to 1} f(x) =$

b) $\lim_{x\to 1+} f(x) =$

c) $\lim_{x\to 1-} f(x) =$

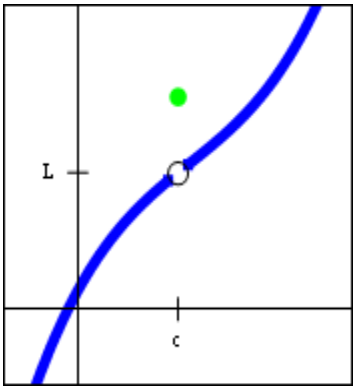
d) $\lim_{x\to \infty} f(x) =$

88. $\lim_{x\to 0} \frac{\sqrt{3-x}-\sqrt{3}}{x}$

89. $\lim_{x\to \infty} \frac{x^2-4x+15}{3x+4x^2}$

90. $\lim_{x\to -\infty} \frac{6x^3-10}{4x+1}$

91. The following is a graph of f(x):



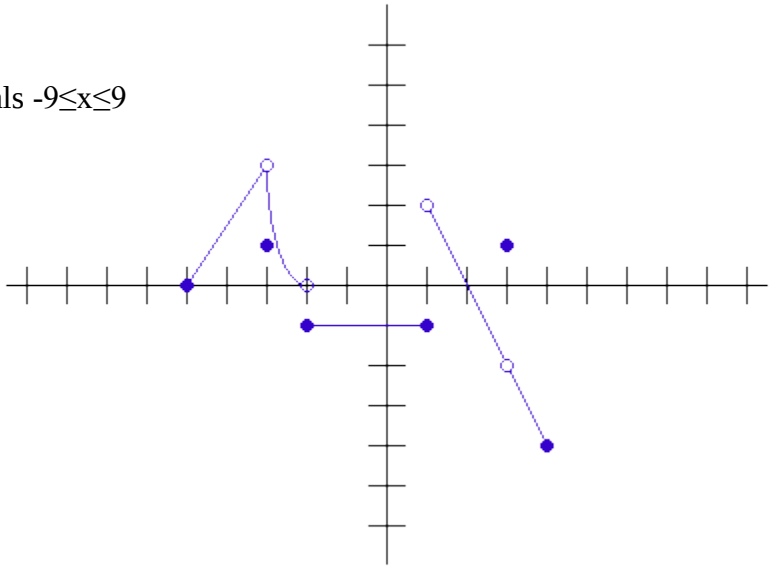
a) $\lim_{x \to c} f(x)$

b) Does $\lim_{x \to c} f(x)$ = f(c)?

92. $\lim_{x \to 9} \sin(\frac{\pi}{18} x)$

93. $\lim_{x \to 3} f(x)$ such that $f(x) = \begin{cases} x^2 + 1, & x \leq 3 \\ 7x - 3, & x > 3 \end{cases}$

94. Pictured is a graph of f(x) on the intervals -9≤x≤9



a) $\lim_{x \to 0} f(x) =$

b) $\lim_{x \to 3} f(x) =$

c) $\lim_{x \to -3} f(x) =$

d) $\lim_{x \to -2} f(x) =$

95. For the following problems, $g(x) = \sqrt{x^2 - 36}$

a) $\lim_{x \to 6} g(x) =$

b) $\lim_{x \to 6^+} g(x) =$

c) $\lim_{x \to 6^-} g(x) =$

96. $\lim_{x\to\infty} \frac{x^2-5}{x^5+x^3+x^2+3} - 5$

97. $\lim_{x\to 3} x$

98. $\lim_{x\to\infty} c^x$

99. $\lim_{x\to 3} \frac{5x^2 - 8x - 13}{x^2 - 5}$

100. $\lim_{x\to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x^3 + 8}$

101. $\lim_{x\to 2} \frac{x^2-4}{x^2-x-2}$

102. $\lim_{x\to\infty} \frac{14x-3}{3x^5-2x^4-3x-1}$

103. $\lim_{x\to 1} \frac{x^3-1}{x-1}$

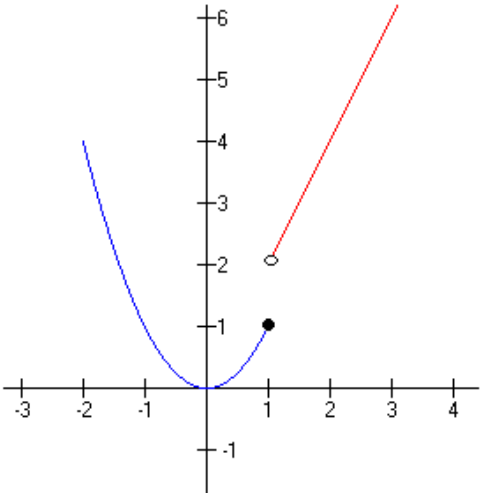
104.

a) $\lim_{x\to 1^-} f(x) =$

b) $\lim_{x\to 1^+} f(x) =$

c) $\lim_{x\to 1} f(x) =$

d) $f(1) =$



Section 8 – Solving Equations

Solve without a calculator

105. $x^4 - 6x^2 + 8 = 0$

106. $2\cos(x) + 1 = 0$

107. $3\csc^2 x - 4 = 0$

108. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

109. $\sin(2x) = -1$

110. $\cos\left(\frac{x}{3} - \frac{\pi}{4}\right) = \frac{1}{2}$

111. $\log(x) + \log(x-1) = \log 2$

112. $\log_4 2 - \log_4 y = 1$

113. $3^{x+1} = 81$

114. $e^{-x^2} = (e^x)^2 * \frac{1}{e^3}$