Summer Packet
List of Helpful Resources

Compiled by Last Year's Students
If these are not sufficient feel free to find other sources of information

Types of Graphs

- http://www.hippocampus.org/Calculus%20%26%20Advanced%20Math

  (click on "Calculus AB for AP" in the left hand column, then select the different topics from the right column.)

Inverse Functions

- http://www.khanacademy.org/math/algebra-functions/v/introduction-to-function-inverses

Logarithms and Exponentials

Exponential and Logarithmic Functions:

$\log_a x = b$
$a^b = x$

$\ln x = b$
$e^b = x$

Change of base formula:

$\log_a x = \frac{\ln x}{\ln a}$

Laws of Logarithms:

$\log_a(xy) = \log_a x + \log_a y$ $\log_a(\frac{x}{y}) = \log_a x - \log_a y$ $\log_a(x^r) = r\log_a x$

Laws of Exponents:

$a^{x+y} = a^x a^y$ $a^{x-y} = \frac{a^x}{a^y}$ $(a^x)^y = a^{xy}$ $(ab)^x = a^x b^x$

$a^{-x} = \frac{1}{a^x}$ $(\frac{a}{b})^x = \frac{a^x}{b^x}$ $a^{1/x} = \sqrt[x]{a}$ $a^{x/y} = \sqrt[y]{a^x} = (\sqrt[y]{a})^x$

$\sqrt[x]{ab} = \sqrt[x]{a}\sqrt[x]{b}$ $\sqrt[x]{\frac{a}{b}} = \frac{\sqrt[x]{a}}{\sqrt[x]{b}}$

Cancellation Equations:

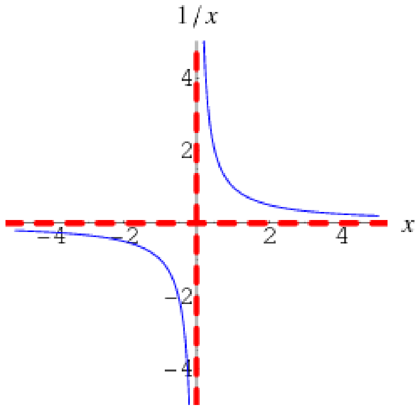
$\log_a(a^x) = x$ $a^{\log_a x} = x$ $\ln e^x = x$ $e^{\ln x} = x$

http://www.purplemath.com/modules/logs.htm

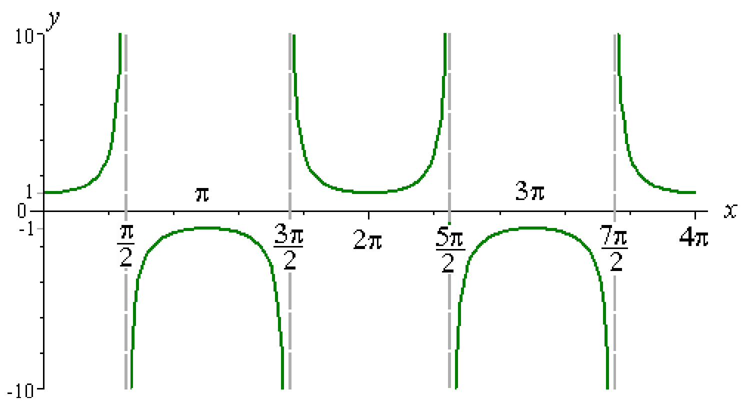
<u>**Characteristics of Rational Functions**</u>

Asymptotes

What are they?
An **asymptote** is a straight horizontal, vertical, or diagonal line that is approached when one of the variables in a given curve goes to infinity.



Vertical Asymptote

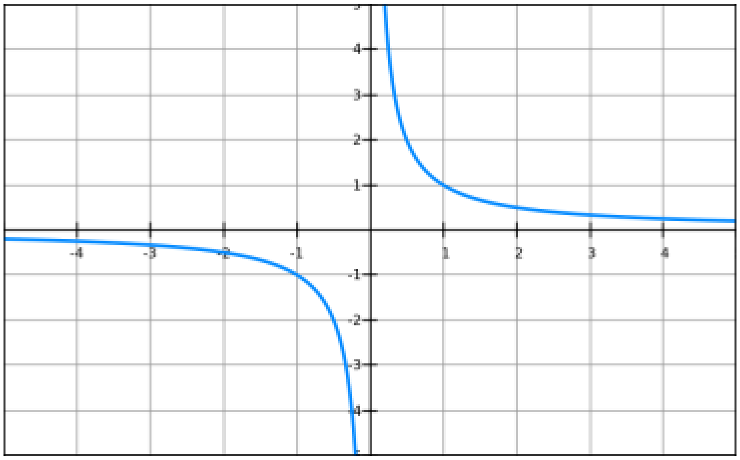


- Where y approaches infinity from both sides of an x-value
- Occur when the denominator of a fraction of a function equals 0
- A Y-value **cannot** exist at a point on the x axis that has an asymptote (in other words, the graph cannot touch the asymptote)

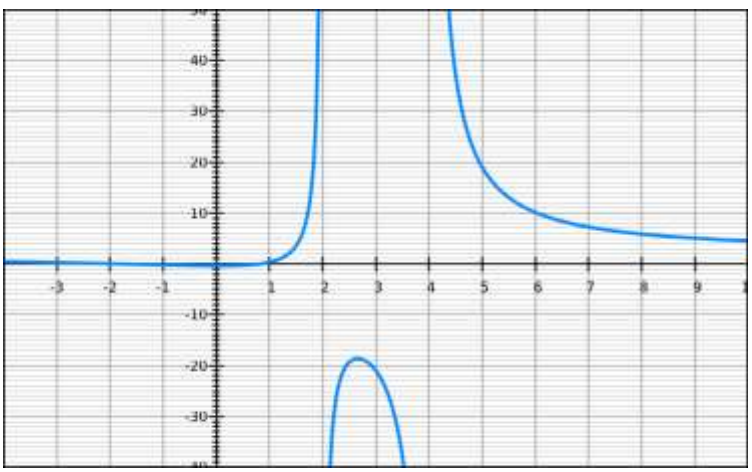
How do you find a vertical asymptote?

http://www.youtube.com/watch?v=_qEOZNPce60&safety_mode=true&persist_safety_mode=1&safe=active

When given an equation, such as $y = \frac{1}{x}$ find the x-value(s) where the denominator equals 0. For this function, the denominator equals o when x=0. When you graph the equation, you will see that at x=0 y approaches infinity from both sides but does not ever exist at the point.



Now, given the function $y = \frac{2x^2+2x-3}{x^2-6x+8}$ factor the denominator to figure out where it equals 0. Here, asymptotes occur at x=4 and x=2, shown below



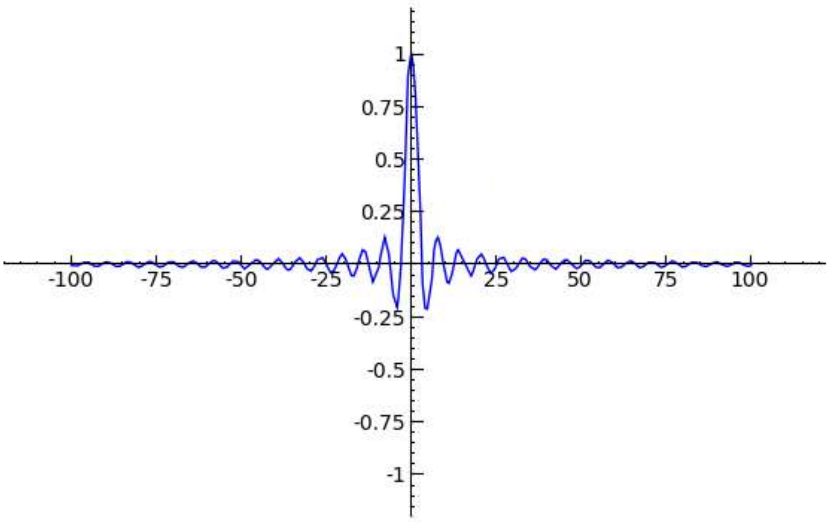
Vertical asymptote vs hole

- A hole occurs at a point where an x value makes both the numerator and denominator 0
- To find where the hole is, factor the equation to its simplest form. The graph will take the shape of this new function, with a hole where the factored out part equals 0.

Example: In the equation $y = \frac{x^2-4x+4}{x+2}$, the numerator becomes (x+2)(x-2). The two (x+2)'s cancel out, leaving the function as y=x-2. Therefore, at x=2, there will be a hole. To find out the y value, plug in x=2 into the new equation, to find that the hole is at (2,0)

Horizontal Asymptote

A **Horizontal Asymptote** is the y-value that a graph approaches as x approaches positive or negative infinity



- The y value at which the graph "levels out" as x gets further from the origin
- A graph can touch or cross a horizontal asymptote, as shown above- the horizontal asymptotes are y=0, despite the graph crossing the value many times.

How do I find the horizontal asymptote?

http://www.youtube.com/watch?v=c-yK2hUnSB0&safety_mode=true&persist_safety_mode=1&safe=active

To find the horizontal asymptote of a function as x approaches positive infinity, find the limit of the function. Using the rules for limits of rational functions the limit of $y = \frac{2x^2+2x-3}{x^2-6x+8}$ is y=2. Therefore, as shown in the graph above, the graph gets closer and closer to y=2 as x approaches infinity.

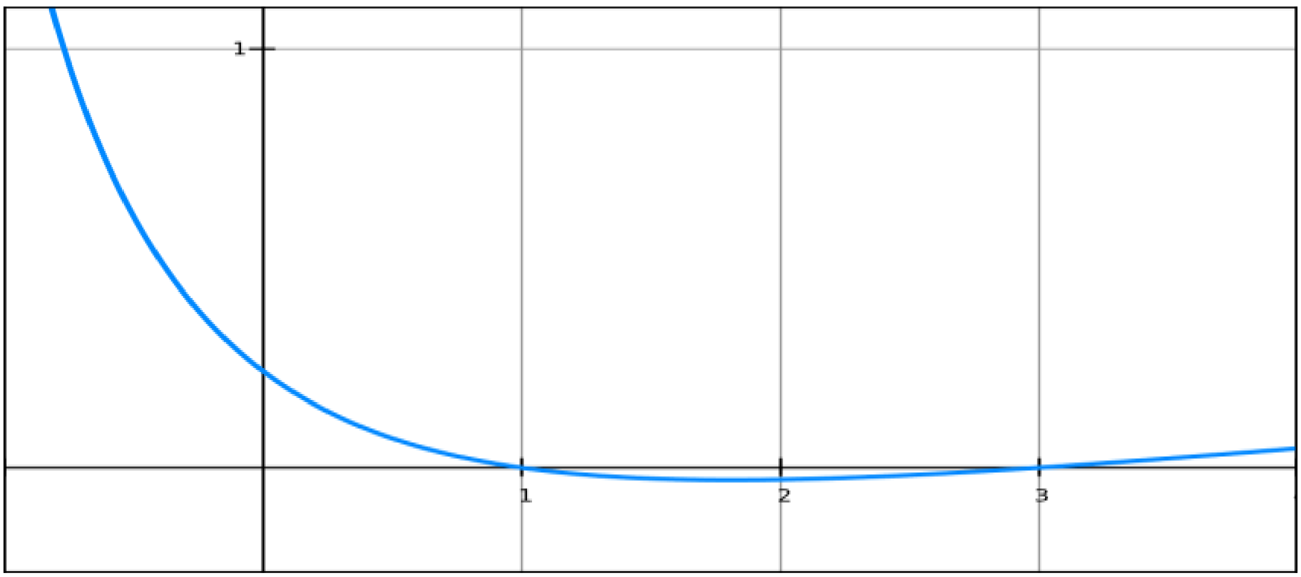
To find the horizontal asymptote of a function as x approaches negative infinity, plug in large negative numbers for x and see what the graph does. For the function, $y = 5^x$, as x becomes more and more negative, the function becomes a smaller and smaller function. Since it gets closer to 0, the horizontal asymptote as x approaches negative infinity is y=0.

Graph intersections

Finding where the graph intersects the x-axis:

- This is the point where y=0, since it intercepts the x axis. Therefore, set y equal to 0 in the function and solve for x.
- In a rational fraction function, this will mean that the x-intercepts are where the **numerator** equals 0.

Take the function $y = \frac{x^2-4x+3}{x^2+9x+13}$. When the numerator equals 0, x= 3 and 1. At these x values, the graph intersects with the x-axis.



Finding where the graph intersects with the y-axis:
The graph intersects the y-axis when x equals 0. Therefore, set x equal to 0 in the function and solve for y. At this y value, the function intersects with the y-axis. For $y = \frac{x^2-4x+3}{x^2+9x+13}$, when x=0, y=3/13. The function intersects the y axis at y=3/13.

Domain and Range (click for video)

Domain/Range with asymptotes

With a **vertical asymptote**, since the function can't cross it, the domain must reflect that the graph does not have a value at that x value. If a graph has a vertical asymptote of x=2, the domain would be
(-∞, 2)U(2, ∞)
In a graph with a horizontal asymptote, the range consists of every y-value that exists in the function. In $y = \sqrt{\frac{1}{x}}$ since the horizontal asymptote is y=0, the range is (0, ∞)

Simplifying Algebraic Expressions

- http://library.thinkquest.org/20991/alg2/frace.html

- http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut11_complexrat.htm

Trigonometry

Trigonometric Identities for AP Calculus

Reciprocal identities

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u} \quad \tan u = \frac{1}{\cot u}$$

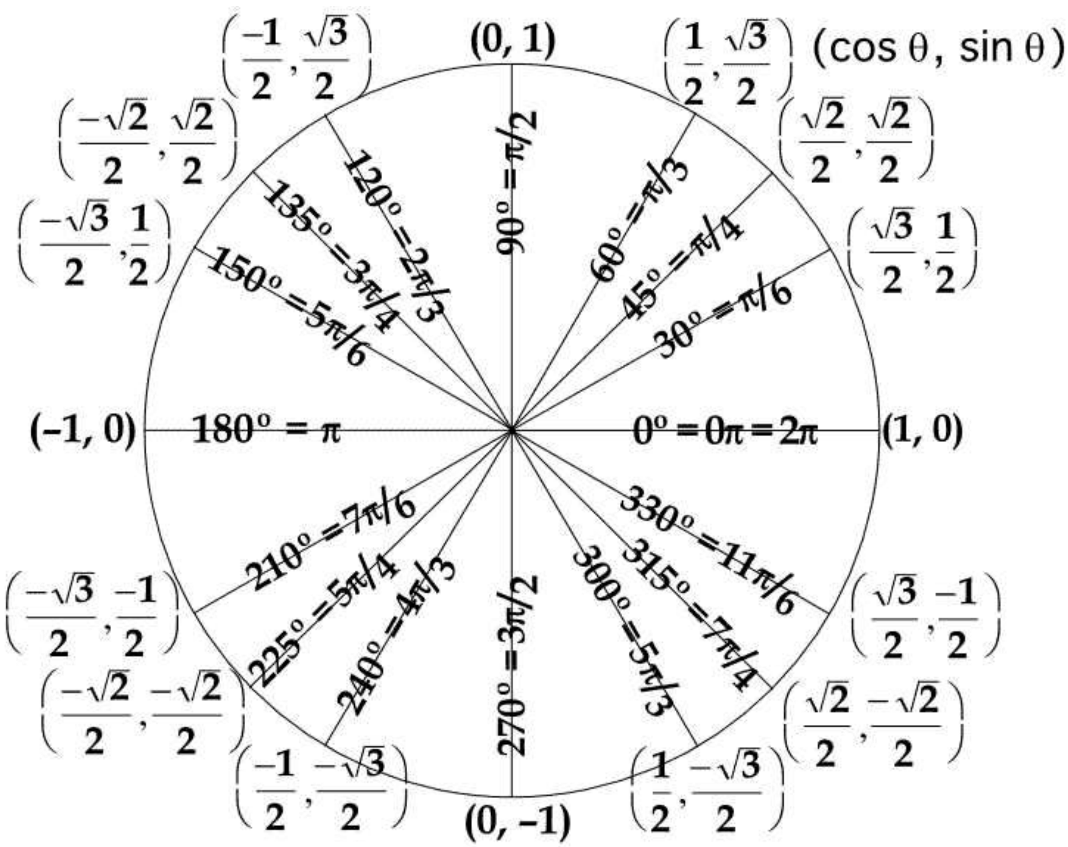
$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u} \quad \cot u = \frac{1}{\tan u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \quad 1 + \tan^2 u = \sec^2 u \quad 1 + \cot^2 u = \csc^2 u$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$



- Quiz Yourself on Unit Circle

http://cs.bluecc.edu/calculus/TrigTools/Files/UnitCircleValues/FunctionValues.htm

Limits

Intro:

- "**limit**" is used to describe the value that a function or sequence "approaches" as the input or index approaches some value.[
- Suppose f(x) is a real-valued function and c is a real number. The expression $\lim_{x \to c} f(x) = L$ means that f(x) can be made to be as close to L as desired by making x sufficiently close to c. In that case, the above equation can be read as "the limit of f of x, as x approaches c, is L".
- A function is CONTINUOUS if: $\lim_{x \to a} f(x) = f(a)$

- http://www.calculus-help.com/tutorials/

- http://www.khanacademy.org/math/calculus/v/introduction-to-limits

- http://www.mathsisfun.com/calculus/limits-infinity.html